P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
III B.Sc. MATHEMATICS-Semester V (W.e.f. 2018-2019)
Course: Ring Theory & Vector Calculus

Total Hrs. of Teaching-Learning: 75 @ 5 hr/Week **Total credits: 5**

OBJECTIVES:

- To impart knowledge on Ring Theory and its applications.
- To make awareness of the concepts of the transformation between line Integral, Surface Integral and Volume integral.
- To introduce the concepts of geometrical meaning of Gradient, Divergence and Curl

RING THEORY

Unit – I: Rings – I **(15 hrs)**

Definition of Ring and Basic Properties, Boolean Rings, Divisors of Zero and Cancellation Laws in Rings, Integral Domain, Division Ring and Fields, The Characteristic of a Ring – The Characteristic of an Integral Domain, the Characteristic of a Field, Sub Rings and Ideals.

Unit – II: Rings – II **(15 hrs)**

Definition of Homomorphism – Homomorphic Image – Elementary Properties of Homomorphism – Kernel of a Homomorphism – Fundamental Theorem of Homomorphism – Maximal Ideals – Prime Ideals.

VECTOR CALCULUS

UNIT – III: Vector Differentiation **(15 hrs)**

Vector differentiation –Ordinary Derivatives of Vector valued functions, Continuity and Differentiation, Gradient , Divergence, Curl operators, Formulae involving these operators.

UNIT – IV: Vector Integration **(15 hrs)**

Line Integral, Surface Integral, Volume Integrals with examples.

Unit – V: Vector Integration Applications **(15 hrs)**

Gauss Divergence Theorem, Stokes theorem, Green's Theorem in plane and applications of these theoremss.

Additional Inputs: Euclidean Ring definition and Examples.

Prescribed text Book:

A text book of Mathematics, Vol. III, S. Chand & Co.

Books for Reference:

1. Topics in Algebra by I.N.Herstine
2. Abstract Algebra by J. Fralieh, Published by Narosa Publishing house
3. Vector Calculus by Santhi Narayan, Published by S.Chand & Company Pvt. Ltd., New Delhi
4. Vector Calculus by R.Gupta, Published by Laxmi Publications.

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-V, PAPER-V

Unit	TOPIC	V.S.A.Q	S.A.Q(including choice)	E.Q(including choice)	Total Marks
I	Rings – I	01	01	02	22
II	Rings – II	01	01	02	22
III	Vector differentiation	01	01	02	22
IV	Vector integration	01	01	01	14
V	Vector Integration Applications	01	01	01	14
TOTAL		05	05	08	94

E.Q = Essay questions (8 marks)
S.A.Q = Short answer questions (5 marks)
V.S.A.Q = Very Short answer questions (1 mark)

Essay questions : 5 x 8 M = 40
Short answer questions : 3 x 5 M = 15
Very Short answer questions : 5 x 1 M = 05

Total Marks = 60

P.R. Government College (Autonomous), Kakinada
III year B.Sc., Degree Examinations - V Semester
Mathematics: Ring Theory & Vector Calculus
Paper – V (Model Paper w. e. f. 2019-2020)

Time: 2 Hrs 30 Min | Max. Marks: 60M

PART – I

Answer ALL the following questions. Each question carries 1 mark. **5 x 1 = 5 M**

1. Define Boolean Ring.
2. Find Kernel of the Homomorphism $f: Z(\sqrt{2}) \to Z(\sqrt{2})$ defined by $f(m+n\sqrt{2}) = m - n\sqrt{2}, \forall\, m+n\sqrt{2} \in Z(\sqrt{2})$.
3. Find div f, where f = grad($x^3+y^3+z^3-3xyz$).
4. Evaluate $\int_0^1 (e^t\bar{i} + e^{-2t}\bar{j})dt$.
5. State the Green's Identities.

PART – II

Answer any THREE of the following questions. Each question carries 5 marks. **3 x 5 = 15 M**

6. Show that a ring R has no zero divisors if and only if the cancellation laws hold in R.
7. Let R and R' be two rings and $f: R \to R'$ be a homomorphism. Then prove that the Kernel of f is an ideal of R.
8. Prove that div Curl $\bar{f} = 0$.
9. If $\bar{F} = y\bar{i} + z\bar{j} + x\bar{k}$, find the circulation of F round the curve C, where C is the Circle $x^2+y^2=1, z=0$.
10. Evaluate $\oint_C (\cos x . \sin y - xy)\, dx + \sin x . \cos y\, dy$, by Green's theorem, where C is the circle $x^2+y^2=1$.

PART – III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks. **5 X 8 = 40 M**

SECTION – A

11. If E = {0, 1, 2, 3, 4}, then prove that (E, $+_5$, $\times_5$) form a field. Justify your answer.
12. Define the characteristic of a ring. Prove that the characteristic of an integral domain is either a prime or zero.
13. State and Prove fundamental theorem of homomorphism in rings.
14. Show that an ideal U of a commutative ring R with unity is maximal if and only if the quotient ring R/U is a field.

SECTION – B

15. Prove that $\nabla \times (\nabla \times A) = \nabla(\nabla . A) - \nabla^2 A$.

16. Find the directional derivative of $f = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of 2i-j-2k.

17. If $\overline{F} = 4xz\overline{i} - y^2\overline{j} + yz\overline{k}$, evaluate $\int \overline{F}.N\, dS$ where S is the surface of the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0, z = a$.

18. Verify Stokes theorem for $A=(2x-y)\overline{i}-yz^2\overline{j}-y^2z\overline{k}$, where **S** is the upper half surface of the sphere $x^2+y^2+z^2=1$ and **C** is its boundary.

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